

# Does Curvature Affect Vascular Haemodynamic and Shear-Stress?

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Many vascular compartments of interest for diagnostic or therapeutic purpose, such as the coronary arteries, the umbilical vessels, the circle of Willis and the aorta itself are curved and/or coiled, causing the appearance of non axial velocity components (influencing the wall's shear-stress) and changing the pattern of the axial velocity components (for instance, by raising the turbulence threshold) [1]. Current clinical flowmetry techniques are based on the principle that, upon insonifying the vessels with a US beam, a (Doppler shifted) signal is obtained which is proportional to the haematic velocity. In the simplest cases, i.e. when the blood is moving in the direction of the vessel's axis, a linear relationship between the Doppler signal and the blood velocity is expected. However also the non-axial components of blood velocity are expected to contribute to the overall Doppler signal, in curved structures [2]. It is therefore important to quantify how vessel's curvature can affect the hemodynamical results and the clinical diagnosis. In order to investigate our hypothesis, we performed "in vitro" measurements by insonifying a curved tube (which simulates a blood vessel) with a probe connected to a US Doppler equipment (Doptek 9000, Seagull, UK). The US power spectra of the shift Doppler so obtained were compared with those predicted by a numerical simulation accounting for the curvature effects. A good agreement between experimental and theoretical curves was observed.

The numerical simulation, however, reconstructs at present only a stationary fluid motion in laminar regime, assuming that the ratio of the tube radius  $a$  and the curvature radius  $R$  is small,

$$\left( \lambda = \frac{a}{R} \ll 1 \right),$$

so that the governing equations have analytical solution [3, 4]. This therefore sets a limit to the general applicability to blood vessels, and the simulation procedure needs further refinements to account for greater  $\lambda$  values, possibly by extension to next approximations.

Fig. 1 shows the profile of the non-axial velocity components in the US beam direction predicted by the numerical model in our experimental conditions, corresponding to  $\lambda = 0.05$ .

The present investigation gives some useful indication about the accuracy of Doppler investigation on curved vessels, i.e. the relative error predicted at any given beam-vessel angle, but more valuable practical consequences will

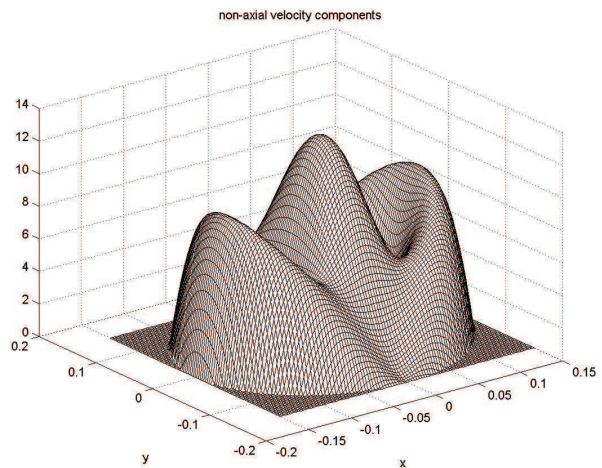


Fig. 1.

derive from the applications of the model (work in progress) to non stationary flows analysis, (as in arterial haemodynamics) and to the assessment of the effects of vessels curvature on the Doppler resistive and pulsatility indices. This research will also allow a better understanding of the biological effects of pulsatile flows in curved vessels on the vascular endothelium.

Being the shear stress defined as:

$$\tau_{w_0} = -\frac{a}{2} \frac{\partial p_0}{\partial x}$$

the model would predict, in a curved vessel and in steady conditions:

$$\tau_w = -\frac{a}{2} \frac{\partial p_0}{\partial x} \left[ 1 + \frac{1}{4} \lambda \sin \theta \left( 1 - \frac{1}{2\mu^2} \left( \frac{\partial p_0}{\partial x} \right)^2 \frac{25a^6}{11520\nu} \right) \right]$$

and in pulsatile conditions:

$$\tau_w = \mu \frac{A_n m a}{2\rho c} \left\{ \frac{\frac{\partial J_1(\alpha_0)}{\partial r'} \left[ \frac{\alpha_0}{a} - 1 \right]}{J_1(\alpha_0)} - \frac{\frac{1}{a} J_0(\alpha_0) + \left[ 1 - \frac{1}{a} \right] J_1(\alpha_0)}{J_0(\alpha_0)} \right\} \sin \theta e^{i\omega \left( t - \frac{z}{c} \right)}$$

An inhomogeneous shear-stress on the wall endothelial cells (depending on the angle  $\theta$ ) is predicted. This would possibly influence the cell morphology and thickness on the vessel wall, suggesting a focused accurate hystological investigation.

### References

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